Probabilistic Fatigue and Damage Tolerance Analysis for General Aviation

Probabilistic fatigue and damage tolerance tool for the Federal Aviation Administration to perform risk analysis

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Outline

- Smart Program Overview
- Smart|LD Capabilities
  - Methodology Review
  - Example Problem
- Smart|DT Capabilities
  - Methodology Review
    - Load Generation/EVD
    - Single Flight/Cumulative Total POF
    - Code Capabilities Flowchart
    - Crack Growth (Master Curve, Kriging, Software Direct Link)
  - Inspection and Repair
    - Recursive Probability Integration for Monte Carlo
    - Inspection and Repair for Numerical Integration
- Example Problem
- GUI
- Current and Future Work
Program Overview

2007-2011
Phase I
Probabilistic Fatigue Analysis for Small Airplanes (SMARTLD)

- Safe-life Approach
  - Prob. Life distribution
  - Hazard Rate
  - Sensitivity Analysis

2009-2013
Phase II
Probabilistic Damage Tolerance Analysis for Small Airplane (SMARTDT)

- SFPOF, Hz, CTPOF
  - Inspection/Repair Effect
  - Sensitivity Analysis

2012-2016
Phase III
Probabilistic Fatigue Management Program for General Aviation

- Develop experience and familiarity with probabilistic approaches within engineering personnel that design, manufacture and maintain general aviation aircraft.
- Verification with in-service findings.
- Develop a Probabilistically-based fatigue management plan (PFMP) for general aviation.
SmartLD Capabilities

- **Loading Generation**
  - Computed from exceedance curves (Internal library and user exceedance option) – Weighted usage available.
  - Flight Duration and Velocity/weight matrices, Design load limit factors, one-g stress, and ground stress as user input.
  - User spectra (Afgrow format)

- **Damage accumulated using Miner’s rule**
  - Safe-Life calculations (in # of flights and # of hours) using Monte Carlo sampling
  - Accumulated damage calculation based on the user number of flight hours.
  - Probability of failure computed using MC sampling

- **Multiple random variables**
  - Library of exceedance curves (weighted mix ok) – Option for user input exceedance.
  - Flight duration, a/c velocity, one-g stress, and ground stress
  - PSN curve constructed from constant amplitude tests – Option for user input PSN
  - Sink Rate
  - Random damage coefficient.
  - Stress Severity Factor (SSF) option

- **Text output files showing Monte Carlo results**
- **Sensitivities computed using correlation and scatter plots**
- **Life distribution and hazard rate calculation**
- **Standard Fortran 95/03, Unix and Windows**
- **GUI**
Risk Methodology

Methodology Summary

- Probabilistic (determined from regression modeling of constant amplitude tests)
- Probabilistic (joint PDF, user defined)
- Probabilistic (joint PDF, user defined)
- Deterministic (Design Scans Allowed)
- Probabilistic (Lognormal)
- Exceedance Curves
- Load Limit Factors
- Monte Carlo Simulation
- Simulated Damage
- Damage Index
- Miner’s Rule
- Miner’s Coefficient
- Life
- Sensitivities
- Sink Rate
- Probabilistic (Weibull or Normal – fit to variable amplitude tests)
Methodology

SMART LD
SMall Aircraft Risk Technology – Linear Damage Analysis
**Damage Methodology (Safe Life)**

1. **Aerobatic - High 1g Stress - All Stages**

2. **Damage Index**
   - $D_i$
   - $S_n$

3. **Miner’s Rule**
   - $d_{current} = d_c$
   - $d_{current} = d_{previous} + 1/N_{ij}$

4. **Yes Next Cycle**
   - $d_c < d_i$

5. **No, Next MC. Record Flights to Failure**

6. **Damage Accumulation**

- $(S_1, S_2)_{ij}$
- i Cycle from Spectrum j

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**UTSA ROADRUNNERS**
Hours Methodology (Current-Future Risk)

(S_{1}, S_{2})_{ij}
i Cycle from Spectrum j

S

S

S

S

\bar{\mu}

S_{2}

\text{Flying Hours < User Fly Hours}

\text{Yes Next Cycle}

\text{No, Next MC Count failure}

\text{Damage Accumulation}

N_{fj}

N

N

\text{Yes}

\text{No, Next MC}

d_{\text{current}} = d_{c}
= d_{\text{previous}} + 1/N_{fj}

\text{d}_{c} < \text{d}_{j}
<table>
<thead>
<tr>
<th>Variable</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gust/Maneuver Load Exceedances</td>
<td>Probabilistic: (Lognormal)</td>
</tr>
<tr>
<td>Aircraft Velocity and Flight Duration</td>
<td>Probabilistic: (Joint PDF with Correlated Variables)</td>
</tr>
<tr>
<td>Maneuver Load Limit Factors</td>
<td>Deterministic</td>
</tr>
<tr>
<td>Gust Load Limit Factors</td>
<td>Deterministic</td>
</tr>
<tr>
<td>Ground/One-g Stress and Flight Duration</td>
<td>Probabilistic: (Joint PDF with Correlated Variables)</td>
</tr>
<tr>
<td>Sink Rate</td>
<td>Probabilistic</td>
</tr>
<tr>
<td>P-S-N</td>
<td>Probabilistic (Determined from regression modeling of constant amplitude tests)</td>
</tr>
<tr>
<td>SSF</td>
<td>PSN Curves (Probabilistic)</td>
</tr>
<tr>
<td></td>
<td>User Input/ Direct Input (Deterministic)</td>
</tr>
<tr>
<td>Miner’s Damage Index</td>
<td>Probabilistic (Weibull or Normal Distribution– fit to variable amplitude tests)</td>
</tr>
</tbody>
</table>
**Stress Life Curves**

Risk Methodology

**FAA AC-23-13A**

\[ \log(N) = A + B \cdot \log(\text{Seq} + C) + Z \cdot \text{Stdev} \]

\[ \text{Seq} = S_{\text{max}} \cdot (1 - R)^D \]

\[ E = \text{Endurance limit} \]

\[ Z \sim N(0,1) \]

*** SN PARAMETERS ***

- **A** = 11.3196
- **B** = -5.4083
- **C** = 0.0
- **D** = 0.0
- **E** = 0.0
- **Stdev** = 0.5

**ASTM E739-91 & Polynomial**

Test Data

- **Test Data**
- **ASTM**
- **Polynomial**

Different Configurations

- Open Hole
- Filled Hole
- Load Transfer
Example Problem

SMART LD
SMall Aircraft Risk Technology – Linear Damage Analysis
### Example

High performance single-engine airplane with 4,000 pounds of maximum take off

<table>
<thead>
<tr>
<th>Variable</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gust/Maneuver Load exceedances</td>
<td>Probabilistic exceedances curves for Single Engine Unpressurized Executive Usage</td>
</tr>
<tr>
<td>Sink Rate</td>
<td>Probabilistic sink rate</td>
</tr>
<tr>
<td>Design Maneuver Load Limit Factors</td>
<td>+3.41, -1.41</td>
</tr>
<tr>
<td>Design Gust Load Limit Factors</td>
<td>3.80, -1.52</td>
</tr>
<tr>
<td>One g stress</td>
<td>+6,550</td>
</tr>
<tr>
<td>Ground Stress</td>
<td>-1,987</td>
</tr>
<tr>
<td>Aircraft Velocity</td>
<td>153</td>
</tr>
<tr>
<td>Damage Index</td>
<td>Normal distribution with mean 1.0 and standard deviation 0.1</td>
</tr>
<tr>
<td>SN Curve</td>
<td>AC23, PSN ASTM</td>
</tr>
</tbody>
</table>
Example
High performance single-engine airplane
with 4,000 pounds of maximum take off

### Flight length and Velocity Matrix

<table>
<thead>
<tr>
<th>Dur/Vel</th>
<th>0.80</th>
<th>0.85</th>
<th>0.90</th>
<th>0.95</th>
<th>1.00</th>
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</thead>
<tbody>
<tr>
<td>0.50:</td>
<td>0.05</td>
<td>0.05</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>0.60:</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>0.70:</td>
<td>0.10</td>
<td>0.00</td>
<td>0.05</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>0.80:</td>
<td>0.15</td>
<td>0.00</td>
<td>0.05</td>
<td>0.05</td>
<td>0.10</td>
</tr>
<tr>
<td>0.90:</td>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.90</td>
</tr>
<tr>
<td>1.00:</td>
<td>0.25</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.90</td>
</tr>
<tr>
<td>1.10:</td>
<td>0.15</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.95</td>
</tr>
<tr>
<td>1.20:</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.95</td>
</tr>
</tbody>
</table>

### Flight length and Weight Matrix

<table>
<thead>
<tr>
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<th>0.85</th>
<th>0.90</th>
<th>0.95</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50:</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.20</td>
<td>0.80</td>
</tr>
<tr>
<td>0.60:</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.20</td>
<td>0.80</td>
</tr>
<tr>
<td>0.70:</td>
<td>0.10</td>
<td>0.00</td>
<td>0.00</td>
<td>0.15</td>
<td>0.85</td>
</tr>
<tr>
<td>0.80:</td>
<td>0.15</td>
<td>0.00</td>
<td>0.00</td>
<td>0.15</td>
<td>0.85</td>
</tr>
<tr>
<td>0.90:</td>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.90</td>
</tr>
<tr>
<td>1.00:</td>
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<td>0.00</td>
<td>0.00</td>
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<tr>
<td>1.10:</td>
<td>0.15</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.95</td>
</tr>
<tr>
<td>1.20:</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.95</td>
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</table>
## Detailed Output Info

### Input Variables

<table>
<thead>
<tr>
<th>Run</th>
<th>Flight Duration</th>
<th>n/C</th>
<th>Sink Rate</th>
<th>Damage Coefficient</th>
<th>Hertz Factor</th>
<th>Run Factor</th>
<th>One-Way Stress</th>
<th>Ground Stress</th>
<th>FSR</th>
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<tbody>
<tr>
<td>1</td>
<td>0.60</td>
<td>153.0</td>
<td>0.0937</td>
<td>0.0042</td>
<td>-3.3319</td>
<td>0.0519</td>
<td>6450.0</td>
<td>-1907.0</td>
<td>3.08</td>
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<tr>
<td>2</td>
<td>1.00</td>
<td>153.0</td>
<td>1.0019</td>
<td>0.0048</td>
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<td>6450.0</td>
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<tr>
<td>3</td>
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<td>153.0</td>
<td>0.0937</td>
<td>0.0042</td>
<td>-3.3319</td>
<td>0.0519</td>
<td>6450.0</td>
<td>-1907.0</td>
<td>3.08</td>
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<td>0.0937</td>
<td>0.0042</td>
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<td>6450.0</td>
<td>-1907.0</td>
<td>3.08</td>
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<td>0.0937</td>
<td>0.0042</td>
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<td>0.0519</td>
<td>6450.0</td>
<td>-1907.0</td>
<td>3.08</td>
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<tr>
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<td>0.0042</td>
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<td>0.0042</td>
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<td>6450.0</td>
<td>-1907.0</td>
<td>3.08</td>
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<td>8</td>
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<td>0.0042</td>
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<td>-1907.0</td>
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<tr>
<td>9</td>
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<td>0.0042</td>
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<td>0.0519</td>
<td>6450.0</td>
<td>-1907.0</td>
<td>3.08</td>
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<td>1.00</td>
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<td>0.0042</td>
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<td>0.0519</td>
<td>6450.0</td>
<td>-1907.0</td>
<td>3.08</td>
</tr>
</tbody>
</table>

### Percent Damage

<table>
<thead>
<tr>
<th>Run</th>
<th>Percent Damage</th>
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</thead>
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<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
<td>0.0000</td>
</tr>
<tr>
<td>3</td>
<td>0.0000</td>
</tr>
<tr>
<td>4</td>
<td>0.0000</td>
</tr>
<tr>
<td>5</td>
<td>0.0000</td>
</tr>
<tr>
<td>6</td>
<td>0.0000</td>
</tr>
<tr>
<td>7</td>
<td>0.0000</td>
</tr>
<tr>
<td>8</td>
<td>0.0000</td>
</tr>
<tr>
<td>9</td>
<td>0.0000</td>
</tr>
<tr>
<td>10</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

### Hz. Fn

<table>
<thead>
<tr>
<th>Hours/Flights-to-Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0012</td>
</tr>
<tr>
<td>12.95</td>
</tr>
<tr>
<td>256.09</td>
</tr>
<tr>
<td>0.0000</td>
</tr>
<tr>
<td>0.0000</td>
</tr>
<tr>
<td>0.0000</td>
</tr>
<tr>
<td>0.0000</td>
</tr>
<tr>
<td>0.0000</td>
</tr>
<tr>
<td>0.0000</td>
</tr>
<tr>
<td>0.0000</td>
</tr>
</tbody>
</table>

### Detailed output per MC run
## Safe-life Results

### 20,000 Monte Carlo Samples

<table>
<thead>
<tr>
<th></th>
<th>95% CONFIDENCE BOUND</th>
<th>MEAN</th>
<th>95% CONFIDENCE BOUND</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC-23</td>
<td>41,109</td>
<td>41,277</td>
<td>41,445</td>
</tr>
<tr>
<td>ASTM</td>
<td>46,043</td>
<td>46,227</td>
<td>46,043</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>95% CONFIDENCE BOUND</th>
<th>STANDARD DEVIATION</th>
<th>95% CONFIDENCE BOUND</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC-23</td>
<td>11,998</td>
<td>12,116</td>
<td>12,236</td>
</tr>
<tr>
<td>ASTM</td>
<td>13,180</td>
<td>13,309</td>
<td>13,441</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Probability</th>
<th>Hours-to-Failure AC23</th>
<th>Hours-to-Failure ASTM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>40,445</td>
<td>44,343</td>
</tr>
<tr>
<td>0.1</td>
<td>26,462</td>
<td>30,332</td>
</tr>
<tr>
<td>0.01</td>
<td>16,314</td>
<td>21,533</td>
</tr>
<tr>
<td>0.001</td>
<td>10,280</td>
<td>16,391</td>
</tr>
<tr>
<td>0.000223</td>
<td>7,247</td>
<td>12,698</td>
</tr>
</tbody>
</table>
### Correlation Sensitivity Analysis wrt HTF

<table>
<thead>
<tr>
<th></th>
<th>FLIGHTS DURATION</th>
<th>FLIGHT SPEED</th>
<th>SINK RATE</th>
<th>DAMAGE COEFFICIENT</th>
<th>GUST FACTOR</th>
<th>MANEUVER FACTOR</th>
<th>ONE-G STRESS</th>
<th>GROUND STRESS</th>
<th>PSN</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC23</td>
<td>0.07</td>
<td>-0.06</td>
<td>-0.02</td>
<td>0.34</td>
<td>0.86</td>
<td>0.07</td>
<td>-0.30</td>
<td>0.30</td>
<td>0.00</td>
</tr>
<tr>
<td>ASTM</td>
<td>0.00</td>
<td>-0.10</td>
<td>-0.01</td>
<td>0.35</td>
<td>0.66</td>
<td>0.07</td>
<td>-0.28</td>
<td>0.28</td>
<td>0.41</td>
</tr>
</tbody>
</table>

**Plots:**
- AC23 SN Curve
- ASTM SN Curve
- Poly SN Curve
Hazard Function
Example Application

- Fleet of 6 Airplanes.
- Calculate Hazard Next 500 hrs.

The hazard rate is defined as the probability per time unit that a case that has survived to the beginning of the respective interval will fail in that interval.

<table>
<thead>
<tr>
<th>No A/C</th>
<th>Hours</th>
<th>Hz(t)*dt</th>
<th>H(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>22,000</td>
<td>0.002</td>
<td>0.004</td>
</tr>
<tr>
<td>3</td>
<td>30,000</td>
<td>0.01</td>
<td>0.03</td>
</tr>
<tr>
<td>1</td>
<td>45,000</td>
<td>0.0315</td>
<td>0.0315</td>
</tr>
<tr>
<td></td>
<td>Fleet Total Hazard</td>
<td>0.0655</td>
<td></td>
</tr>
</tbody>
</table>

The diagram shows a hazard function graph with the hazard rate on the y-axis and hours on the x-axis. Each airplane is represented by a segment on the graph, indicating the number of airplanes with their respective hours.

- 2 Airplanes with 22,000 hours
- 3 Airplanes with 30,000 hours
- 1 Airplane with 45,000 hours

The total hazard for the fleet is calculated as 0.0655 over the next 500 hours.
Program Overview

2007-2011
Phase I
Probabilistic Fatigue Analysis for Small Airplanes (SMART<sub>LD</sub>)

Safe-life Approach

- Prob. Life distribution
- Hazard Rate
- Sensitivity Analysis

2009-2013
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- Develop experience and familiarity with probabilistic approaches within engineering personnel that design, manufacture and maintain general aviation aircraft.
- Verification with in-service findings.
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Smart|DT Capabilities

- **Loading Generation**
  - Computed from exceedance curves (Internal library and user exceedance option) – Weighted usage available.
  - Flight Duration and weight matrices, Design load limit factors, one-g stress, and ground stress as user input.
  - Stresses and/or flights randomizations
  - Spectrum editing option (Rainflow, rise/fall, Dead band)
  - User-defined spectra (Afgrow format)

- **Extreme Value Distribution**
  - User input, e.g., Gumbel, Frechet, and Weibull.
  - Ultimate/Limit load (deterministic)
  - Computed from exceedance curves, weight matrix, etc. (Gumbel, Frechet, and Weibull)

- **Probability calculations**
  - SFPOF (no survival term)
  - Hazard fn. (with survival term)
  - Cumulative (with survival term)

- **Crack growth**
  - Direct Nasgro link (for all computations – as an option)
  - Extension to Afgrow (Current Work)
  - Through, Corner, Surface crack growth geometry options
  - Master curve for 2D (ai and Kc) interpolation (user input or developed from Nasgro/Afgrow)
  - Kriging for efficient probabilistic fracture analysis

- **Probabilistic methods**
  - Standard Monte Carlo
  - Numerical integration

- **Inspection capabilities**
  - Any number of inspections (arbitrary limit set to 15)
  - Arbitrary repair crack size distribution (lognormal, tabular, deterministic)
  - Arbitrary POD (lognormal, tabular)
  - Deterministic POD
  - User defined probability of inspection
  - Extension to different repairs scenarios (Future Work)

- **Random variables**
  - ai, Kc, Evd – all cases
  - Crack growth parameters, hole diameter, crack aspect ratio

- **Computational implementation**
  - Standard Fortran 95/03 (ifort) - Unix, Windows
  - GUI (Windows)
Material Data
- C and m
- Fracture Toughness
- Yield and Ultimate Stress

Loading Data
- Load Limit Factors
- Exceedance Curves
- Flight Duration Velocity Weight Matrix
- Sink Rate
- Spectrum Length

Inspection Data
- POD
- Repair Crack Size
- Inspection times
- Prob. of Inspecting

Geometry Data
- Initial Crack Size
- Hole Diameter Hole Offset (some models)
POF

SMART<sup>DT</sup>
SMall Aircraft Risk Technology - Damage Tolerance Analysis
The probability-of-failure is the probability that maximum value of the applied stress (during the next flight) will exceed the residual strength $\sigma_{RS}$ of the aircraft component

$$P_f = P_{max} > \frac{K_C}{(a(a_0,t))\sqrt{a(a_0,t)}} = P[\ max > RS]$$

The CDF of the maximum stress in a flight ($F_{EVD}$) can be determined using extreme value theory

$$P_f(t|a,K_C) = 1 \quad F_{EVD} \quad \frac{K_C}{(a(a_0,t))\sqrt{a(a_0,t)}}$$

$$POF(t) = \int_0^\infty F_{EVD} \frac{K_C}{(a(a_0,t))\sqrt{a(a_0,t)}} \frac{f_{a_0}(a_0)f_{K_C}(K_C)}{d_a_0 dK_C}$$

Given these POF calculations, other auxiliary results can be obtained such as the SFPOF (Lincoln and Freudenthal) Cumulative POF and the hazard function.
Loading Generation

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SMall Aircraft Risk Technology - Damage Tolerance Analysis
Randomize taxi loads and split half before the flight and half after the flight, Taxi load can be excluded from the analysis.

Landing and rebound are placed after the flight and before the post taxi.

Randomize gust, maneuver, and GAG.
Smart allows the user to load Afgrow spectra files (.sp3 and .sub). The GUI will read the “.sp3”
EVD Generation

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A critical component is the extreme load per flight. This extreme load is (should be) determined from the same spectrum used for the crack growth.

$$POF(T) = \int_{-\infty}^{\infty} \left[ 1 - F_{EVD} \left( \frac{K_c}{\beta(T) \sqrt{\pi a(T)}} \right) \right] f_{a_0}(a_0) f_{K_c}(K_c) da_0 dK_c$$
Weibull, Frechet, or Gumbel can be written in terms of the Generalized Extreme Value Distribution as

$$F(x) = \exp\left(1 + \frac{x}{\mu/s}\right)^{-1/\xi}$$

- $= 0$ Gumbel
- $> 0$ Frechet
- $< 0$ Weibull

Parameters $(\mu, \sigma, \xi)$ location, scale, and shape define the distribution.
EVD Results

- Single Engine Unpress Basic Inst Usage (Frechet)
- Single Engine Unpress Pers Usage (Gumbel)
- Single Engine Unpress Exec Usage (Weibull)
- Twin Engine Unpress Basic Inst Usage (Gumbel)
- Twin Engine Unpress General Usage (Frechet)
- Pressurized Usage (Frechet)
- Agric Special Usage (Weibull)
- Special Usage Survey (Weibull)
Smart|DT allows the user to input the limit load as EVD input. The limit load behaves as an step function, residual Strength smaller or equal than the limit load has a POF = 1 and, residual Strength bigger than the limit load has a POF = 0.

EVD is set to a deterministic values equal to the airplane limit/ultimate load.
Crack Growth Methods

SMART DT
SMall Aircraft Risk Technology - Damage Tolerance Analysis
Analysis Methods

**Master Curve**
Monte Carlo/
Numerical Integration

- ✔️ Only $a_i$ and $K_c$ Random
- ✔️ Representative Spectrum
- ✔️ Inspection/Repair

**1X - Efficiency**

- ✔️ Hz, SFPOF, CTPOF

**Full Nasgro/Afgrow**
Monte Carlo/
Numerical Integration

- ✔️ Multiple rnd variables
- ✔️ Multiple Spectrum
- ✔️ Inspection/Repair

**1000X - Efficiency**

- ✔️ Hz, SFPOF, CTPOF

**Adaptive Kriging/Nasgro**
Monte Carlo/
Numerical Integration

- ✔️ Multiple rnd variables
- ✔️ Representative Spectrum
- ✔️ Inspection/Repair

**10X - Efficiency**
Methodology
Probabilistic Damage Tolerance for Small Airplanes

- **Geometry**
- **Material Properties**
- **Loading**
- **EIFS**
- **EVD**
- **POD**
- **Adaptive Surrogate Model**

The diagram illustrates the process of loading, leading to EVD and POD, and then to crack size over time with adaptive surrogate models. The loading data is shown in the top left, with subsequent steps involving EVD and POD analysis. The adaptive surrogate model is used to predict crack size over time, with inspirations from EIFS and RS. The material properties and SFPOF/CTPOF/Hz are also considered in the analysis, with sensitivities shown in the bottom right.
An adaptive Kriging surrogate model is used to reduce physics-based crack growth function calls, e.g., AFGROW, FASTRAN, UniGrow

- Applicable to both:
  - POF calculations (residual strength predictions) and inspections (crack growth predictions)

- Adaptive (self correcting):
  - additional crack growth function calls added as needed per user-defined error threshold.

\[
POF(t) = 1 - F_{EVD}(RS(K_C, a_o, C, m, t)) \int f_X(x) dx
\]
Kriging Schematic
Kriging Error Prediction

Compute prediction variance and confidence bounds
The error is calculated based on the Kriging variance and the assumption that $Z(\cdot)$ is Gaussian.

The 95% confidence bound from the prediction value can be computed as

$$A \equiv (A_{LB}, A_{UB}) = (\hat{Z}(x_0) - 1.96\sigma_\epsilon(x_0), \hat{Z}(x_0) + 1.96\sigma_\epsilon(x_0))$$

The error based on the 95% (99%) confidence bound can be computed as

$$\text{error} = \frac{abs(A_{LB} - \hat{Z}(x_0))}{A_{LB}}$$
Kriging Adaptive Model

Generate Initial Random Realization (Training Points)

- Build Kriging RS Response Surface
- Build Kriging Crack Size Response Surfaces

Generate Random Realization of the Random Variables (material, geometric)

- Upgrade Kriging Response Surface
- Run Crack Growth Software

Evaluate Kriging Surrogate Models

- Is the error Acceptable?
  - No
  - Yes: Compute POF

*Only at time of inspections

RS every N (User Define) flights and Crack size according to Inspection Schedule
Residual strength Kriging surfaces are created anew at each time step requested by the user using non-failed realizations. Similarly for crack size estimates.
If an inspection occurs at time $t$, crack size Kriging surfaces are created at each inspection time.
Adaptive Kriging
Multiple Random Variables
Inspections and Repair

SMART<sub>DT</sub>

SMall Aircraft Risk Technology - Damage Tolerance Analysis
Implementation
Monte Carlo

- Weighted sum of possible crack growth paths
- 1 additional path for each inspection

For Each Realization

\[ p_{1,1} = 1 - POD(CS_{1,1}) \]
\[ p_{1,2} = POD(CS_{1,1}) \]
\[ \sum_{j=1}^{2} p_{1,j} = 1.0 \]

\[ p_{2,1} = 1 - POD(CS_{2,1}) \]
\[ p_{2,2} = 1 - POD(CS_{2,2}) \]
\[ \sum_{j=1}^{3} p_{2,j} = 1.0 \]

\[ POF_1 = POF_{\text{ORI}} \cdot p_{1,1} \]
\[ POF_2 = POF_{1^{st} \text{REP}} \cdot p_{1,2} \]
\[ POF_3 = POF_{2^{nd} \text{REP}} \cdot p_{3,2} \]
\[ POF = \frac{1}{2} \sum_{j=1}^{2} POF_i \]
\[ POF = \frac{1}{3} \sum_{j=1}^{3} POF_i \]
After inspection, some cracks are detected and repaired. The post-inspection crack size distribution becomes a combination of a “before” and a “repair” distribution

\[
f_{after}(a) = P_{det}f_{R}(a) + [1 - POD(a)]f_{before}(a)
\]

\[
P_{det} = POD(a)f_{before}(a)da - \% \text{ of cracks detected}
\]

\[
f_{before} - \text{crack size at the time of inspection}
\]

\[
f_{after} - \text{crack size after inspection}
\]
Example Problem

SMART$_{DT}$

SMall Aircraft Risk Technology Damage Tolerance Analysis
### Quantity | Definition
--- | ---
Nasgro Crack Growth Model. | TC03 – Through crack in a hole
**Geometric Variables** | Width = 2.5 in.  
Thickness = 0.09 in.  
Hole Diameter = 0.10 in.  
Hole Offset = 0.5 in.
**Fracture Toughness Distribution** | Normal:  
Mean = 34.8 ksi√in.  
Standard Deviation = 3.9 ksi√in.
**Initial Crack Size Distribution** | Lognormal  
Median = 0.00163 in.  
Mean = ln(median) = -6.420  
Standard Deviation = 1.113
**Extreme Value Distribution (Weibull)** | Location = 5.0, Scale = 10.0, and Shape = 5.0
**Material** | Al-2024
<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Usage</strong></td>
<td>Single Engine Unpressurized Basic Executive Usage</td>
</tr>
<tr>
<td>Design LLF Maneuver</td>
<td>3.8, -1.52</td>
</tr>
<tr>
<td>Design LLF Gust</td>
<td>3.155, -1.155</td>
</tr>
<tr>
<td>Ground Stress (psi)</td>
<td>-4,550</td>
</tr>
<tr>
<td>One-g stress (psi)</td>
<td>7,100</td>
</tr>
<tr>
<td>Flight Length and Velocity Matrix</td>
<td></td>
</tr>
<tr>
<td>Flight Length and Weight Matrix</td>
<td></td>
</tr>
<tr>
<td>Average Velocity (Vno/Vmo (Knots))</td>
<td>165</td>
</tr>
</tbody>
</table>
High Performance Aircraft no Inspection
High Performance Aircraft no Inspection

POF Results 5000 Samples ($a_i$ and $Kc$ Random) 2% Error Threshold

- Full Nasgro Solution
- Adaptive Kriging

Fully Nasgro = 17 hrs 50 min
Adaptive Kriging = 2 hrs 20 min
<table>
<thead>
<tr>
<th>Quantity</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nasgro Crack Growth Model.</td>
<td>TC03 – Through crack in a hole</td>
</tr>
<tr>
<td>Geometric Variables</td>
<td>Width = 2.5 in. Thickness = 0.15 in. Hole Diameter = 0.10 in. Hole Offset = 0.5 in.</td>
</tr>
<tr>
<td>Fracture Toughness Distribution</td>
<td>Normal:</td>
</tr>
<tr>
<td></td>
<td>Mean = 40.0 ksi√in. Standard Deviation = 4.0 ksi√in.</td>
</tr>
<tr>
<td>Initial Crack Size Distribution</td>
<td>Lognormal</td>
</tr>
<tr>
<td></td>
<td>Median = 0.050 in. Mean = ln(median) = -2.995 Standard Deviation = 0.001</td>
</tr>
<tr>
<td>Material</td>
<td>AI-2024</td>
</tr>
</tbody>
</table>
### Commuter Aircraft with Inspections

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Usage</td>
<td>Twin Engine Unpressurized Basic Executive Usage</td>
</tr>
<tr>
<td>Design LLF Maneuver</td>
<td>3.2, -1.5</td>
</tr>
<tr>
<td>Design LLF Gust</td>
<td>3.2, -1.2</td>
</tr>
<tr>
<td>Ground Stress (psi)</td>
<td>-4,000</td>
</tr>
<tr>
<td>One-g stress (psi)</td>
<td>5,100</td>
</tr>
<tr>
<td>Flight Length and Velocity Matrix</td>
<td>Deterministic (1 hr. Duration)</td>
</tr>
<tr>
<td>Flight Length and Weight Matrix</td>
<td>deterministic</td>
</tr>
<tr>
<td>Average Velocity (Vno/Vmo (Knots))</td>
<td>165</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inspection Time</td>
<td>5,000</td>
</tr>
<tr>
<td>Probability of Inspection</td>
<td>1.0</td>
</tr>
<tr>
<td>Probability of Detection</td>
<td>Lognormal</td>
</tr>
<tr>
<td></td>
<td>Median = 0.00390 in.</td>
</tr>
<tr>
<td></td>
<td>Mean = ln(median) = -5.545 in.</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation = 1.113 in.</td>
</tr>
<tr>
<td>Repair Crack Size Distribution</td>
<td>Lognormal</td>
</tr>
<tr>
<td></td>
<td>Median = 0.050 in.</td>
</tr>
<tr>
<td></td>
<td>Mean = ln(median) = -2.995</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation = 0.001</td>
</tr>
</tbody>
</table>
Commuter Aircraft with Inspections

**Probability vs. Stress (KSI)**

- **Twin Engine Unpress General Usage (Frechet)**
  - Location = 8.9955, Scale = 0.7778, Shape = 0.1239
Commuter Aircraft with Inspections
Future & Current Work

**SMART**

**DT**

**SMall Aircraft Risk Technology Damage Tolerance Analysis**
Current/Future Work

✓ Improved sampling methods:

✓ High Performance Computing:
Current/Future Work

✓ Extension to Different repair scenarios

POF

\[ p_{1,1} = P(\text{Detected}) = 1 - POD(CS_{1,1}) \]
\[ p_{1,2} = P(\text{NoDetected}) = POD(CS_{1,1}) \]
\[ \sum_{j=1}^{2} p_{i,j} = 1.0 \]

\[ POF_1 = POF_{\text{ori}} \cdot p_{1,1} \]
\[ POF_2 = POF_{\text{repair}1(CS_{1,1})} \cdot p_{1,2} \cdot p_{CS_{1,1}(\text{small})} \]
\[ POF_3 = POF_{\text{repair}2(CS_{1,1})} \cdot p_{1,2} \cdot p_{CS_{1,1}(\text{medium})} \]
\[ POF_4 = POF_{\text{repair}3(CS_{1,1})} \cdot p_{1,2} \cdot p_{CS_{1,1}(\text{large})} \]

\[ POF = \frac{1}{\text{paths}} \sum_{i=1}^{\text{paths}} POF_i \]

Simple Oversize
Minor Repair (patch)
Major Repair (Replacement)
GUI
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Questions

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