

## Load and Stress Spectrum Generation

During 1962, at the request of the FAA, and upon recommendation of the National Aeronautics and Space Administration (NASA) Committee on Aircraft Operating Problems, the NASA V-G (velocity, normal acceleration)/VGH (velocity, normal acceleration, pressure altitude) General Aviation Program was established [7]. This program recorded gust and maneuver loads, airspeed practices, and other variables for general aviation airplanes. This information provided a data bank for use by airplane designers and evaluators. The program recorded more than 105 airplanes with more than 42,155 hours of VGH data. Tabulated data in exceedance form can be found in the FAA reports [AFS-120-73-2](#) and [AC23-13A](#) and in the “[Probability Basis of Safe-Life Evaluation in Small Planes](#)” paper for probabilistic analysis.

## Maneuver and Gust Loads

As previously discussed, maneuver and gust data were developed under NASA V-G/VGH General Aviation Program. The data for maneuver and gust load are presented as cumulative number of occurrences per nautical mile versus acceleration fraction (an airplane characteristic defined as the incremental normal acceleration divided by the incremental limit factor).

The same methodology is used to generate maneuver and gust load data, so it is sufficient to show how to generate only one of these. The following example shows the procedure to generate gust load.

To calculate the damage due to gust, it is necessary to calculate the airplane load limit factor. The airplane load limit factor is an airplane characteristic determined from the airplane manufacturer.

Data for airplane characteristics used to calculate the load limit factor can be found in the [General Aviation Program](#).

To read the data available in the exceedance curves it is necessary to normalize the gust spectrum. The gust spectrum is expressed in terms of the gust load factor ratio:

$$\frac{a_n}{a_{nLLF}} = \frac{\text{Incremental normal acceleration (delta g)}}{\text{Load limit factor}} \quad (1)$$

To develop the stress spectrum, the code sweeps through the spectrum of acceleration fraction values to account for all the possible loads that an airplane faces during a flight. For this example, only four values were chosen for illustration purposes (0.10, 0.16, 0.22, and 0.28). The values and calculations needed to compute damage due to gust loading are shown in Table 1, with the description in the second column. Steps to calculate gust damage are explained more in detail as follows.

Figure 1 shows how positive and negative acceleration fractions are read, including their corresponding cumulative frequency of exceedance. Occurrence frequency is the difference between two successive values of cumulative frequency. The process starts by generating values from 0.1 through 1.0 of the acceleration

fraction; SMART|LD generates a total of 38 evenly spaced values to account for all the possible loads during a flight. Figure 1 shows positive values of acceleration fraction (0.1 and 0.16). Using these values, the respective cumulative frequency values are read (0.57 and 0.18) with its corresponding negative acceleration fraction at the same cumulative frequency level (-0.12 and -0.18).

To calculate occurrence frequency, the difference between two successive values of cumulative frequency is calculated. In this case, the difference between 0.57 and 0.18 is calculated resulting in 0.39 occurrences per nautical mile. The number of occurrences per hour is calculated by multiplying the cumulative frequency per nautical mile by the aircraft velocity in nautical miles per hour. For this example, the velocity is assumed as 90% of the aircraft design velocity. In this case we multiply 0.39 occurrences per nautical mile by 148.2 nautical miles per hour (90% of the design velocity) resulting in 57.8 occurrences per hour.

The stresses at this occurrence level (57.8) can be calculated by multiplying the acceleration fractions involved (0.13, the average between two successive positive values of 0.1 and 0.16, and -0.12) by the load limit factor (2.155) to obtain delta g. This delta g value can be multiplied by the 1g stress value to obtain the maximum and minimum stress.

Using Equation 2 and Equation 3, the mean and alternating stresses can be calculated, and then the life can be read from Figure 2 ([AC23- deterministic S-N Curve](#)). Table 1 shows the calculation summary for different acceleration fractions.

$$S_m = \frac{[(lg\_stress) + pos\_delta\_g] + [(lg\_stress) + neg\_delta\_g]}{2} \quad (2)$$

$$S_a = [(lg\_stress) + pos\_delta\_g] - S_m \quad (3)$$

where  $S_m$  and  $S_a$  stands for mean stress and alternating stress that are needed to read the AC 23-13A S-N curve.

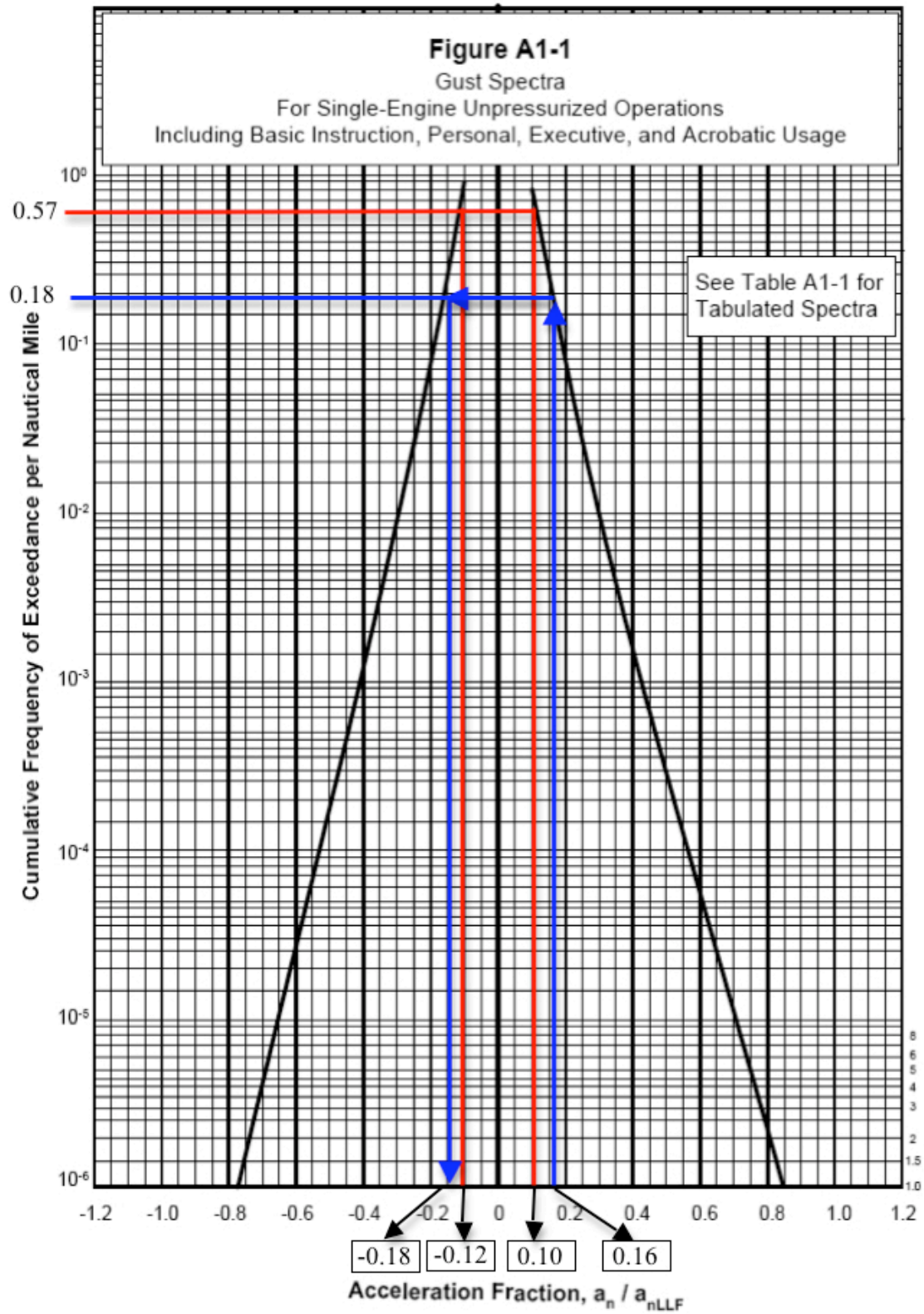


Figure 1. Exceedance Curve for Gust

Gust Damage					
1	From Figure 1 we make a sweep reading the $a_n/a_{nLLF}$ (+), Positive values of gust load factor ratio	0.10	0.16	0.22	0.28
		0.16	0.22	0.28	0.34
2	Calculate the average value from the values in Row 1	0.13	0.19	0.25	0.31
3	From Figure 1 we read the different values of cumulative occurrence of gust per nautical mile at a specific gust load factor.	0.57	0.18	0.052	0.013
4	From Figure 1 we read the $a_n/a_{nLLF}$ (-), negative values of gust load factor ratio at a positive gust load factor (Row 2).	-0.12	-0.18	-0.25	-0.32
5	Frequency per nautical mile (no accumulation) is the difference between two successive values in Row 3.	0.39	0.128	0.039	0.0087
6	Number of gust cycles accumulated per hour(Row 5) x 0.9(Design Cruise Speed,Vc).	57.8	19	5.79	1.292
7	Increment in the stress due to the gusts. To get this value (delta g) multiply Row 2 and Row4 by $a_{nLLF} = 2.155$ (description)	-0.259	-0.388	-0.539	-0.690
		0.28	0.409	0.539	0.667
8	Maximum and minimum delta stress over and below the maximum stress at the critical component (One g Stress) – multiply Row (7) by One g Stress (7410 psi)	-1920	-2870	-4000	-5980
		2070	3030	4000	4940
9	Mean stress (psi). This is calculated as in Equation 2.	7485	7490	7410	7325
10	Alternating stress (psi) calculation. This is calculated as in Equation 3.	1995	2950	4000	5025
11	Endurance cycles, N, read from Figure 2 is (S-N diagram) x10 <sup>6</sup> .	14	3.1	1.05	0.50
12	Damage per hour, Row (6) divided by row (10) x10 <sup>-6</sup>	4.13	6.13	5.52	2.59

Table 1. Gust Damage Calculation

One important concept introduced in Table 1 is one g stress; this stress is measured at the critical location when the airplane is under only the gravitational load. The ground stress is the stress measured at the critical location when the airplane is on the ground.

Having the mean and the alternating stress and using Figure 2 ([Deterministic S-N Curve](#)), the life is calculated at each stress level. The lines in red in Figure 2 correspond to the values of mean and alternating stress used to calculate life for column 3 (first set of acceleration fractions) in Table 1.

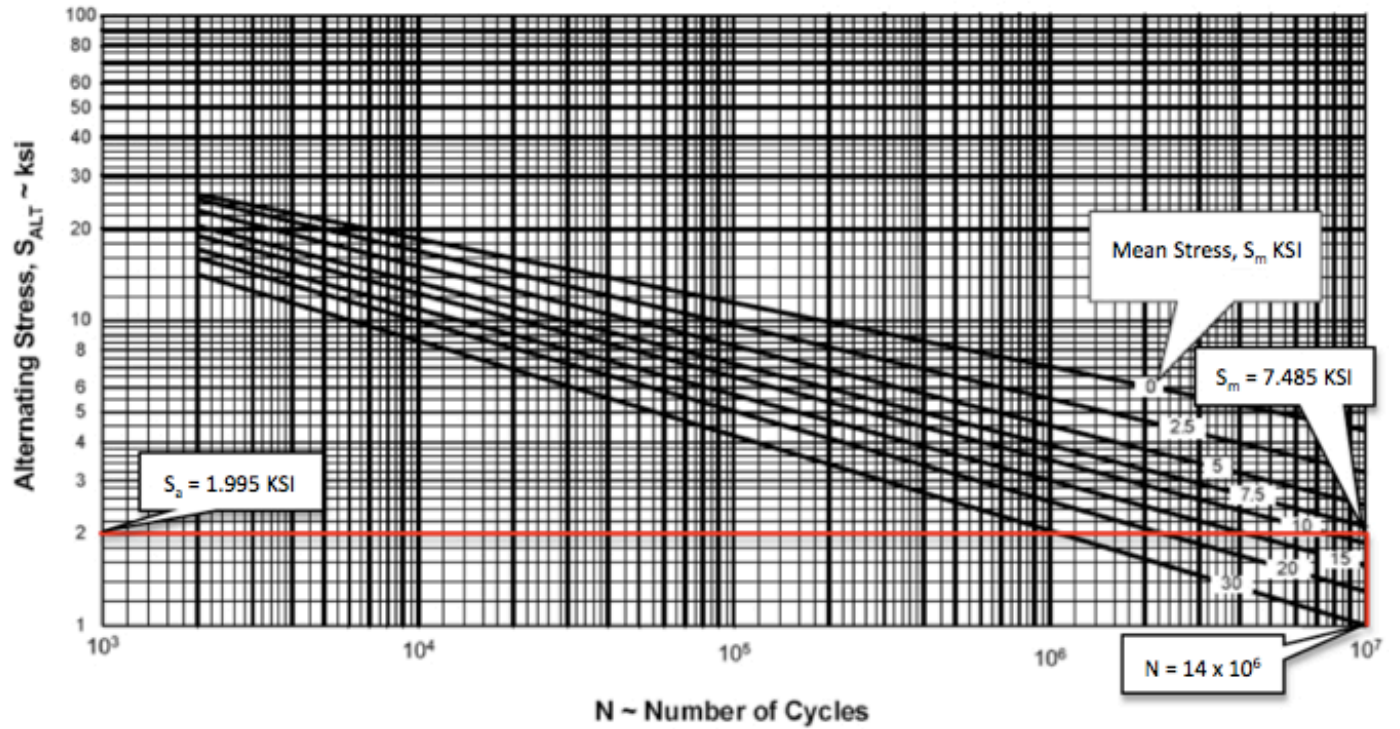


Figure 2. S-N Curves for Aluminum Components [5]

SMART|LD uses Miner's Rule as a damage accumulation rule. Miner's Rule is shown in Equation 4, and can be easily explained as: any load cycle with amplitude  $S_i$  adds to the cumulative damage ( $D$ ), a quantity  $(1/N_i)$ . Here,  $N_i$  denotes the fatigue life under constant amplitude loading with amplitude  $S_i$ , and  $n_i$  is the number of load cycles at this stress value with a variable amplitude loading. Failure is assumed to occur when  $D$  reaches a critical value. More information about  $D$  will be given below in the Miner's Damage Index section.

$$\sum_{i=1}^k \frac{n_i}{N_i} = D = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \dots + \frac{n_i}{N_i} \quad (4)$$

The total damage due to gust loading is the sum of the last row (Row 12) in Table 1; in this case:  $4.13+6.13+5.52+2.59 = 18.37 \times 10^{-6}$  per hour. The values in Row 12 come from Miner's Rule, which is the number of occurrences (Row 6) divided by

the total life ( $N_i$ ) at that stress level (Row 11), so the damage is dimensionless. This damage value is only an example that pertains to the four acceleration fraction levels selected for illustration purposes.

The result from this example is expressed in damage per hour. We obtain the damage-per-flight by multiplying damage per hour by the flight duration. The procedure to calculate maneuver load spectrum and damage is similar; however, the maneuver's exceedance curve must be used.

### **Landing and Rebound Loads**

Using the data presented in FAA report [AC 23-13A](#), an average value of 3.0 feet per second was established for the sink rate velocity. Report [AFS-120](#) presents landing gear drop test data which is shown in Figure 3. With the sink rate velocity information, the load factor ( $g$ ) can be calculated. The equation developed from Figure 3 is shown as follows:

$$g = 0.1877 \cdot (\text{Sink Rate Velocity}) + 1.3422 \quad (5)$$

where 0.1877 is the slope and 1.3422 is the intercept. This equation was developed using linear regression. After calculating the load factor, the maximum and minimum stresses for landing and rebound are calculated with the equations:

$$Landing_{MAX} = \frac{2}{3} (lg\_Stress) \quad (6)$$

$$Landing_{MIN} = (Ground\_Stress) \cdot g \quad (7)$$

$$Rebound_{MAX} = 0.6 \cdot Landing_{MAX} \quad (8)$$

$$Rebound_{MIN} = 0.6 \cdot Landing_{MIN} \quad (9)$$

### Landing Gear Drop Test Data (Assumed 2/3 Lift Acting on Aircraft)

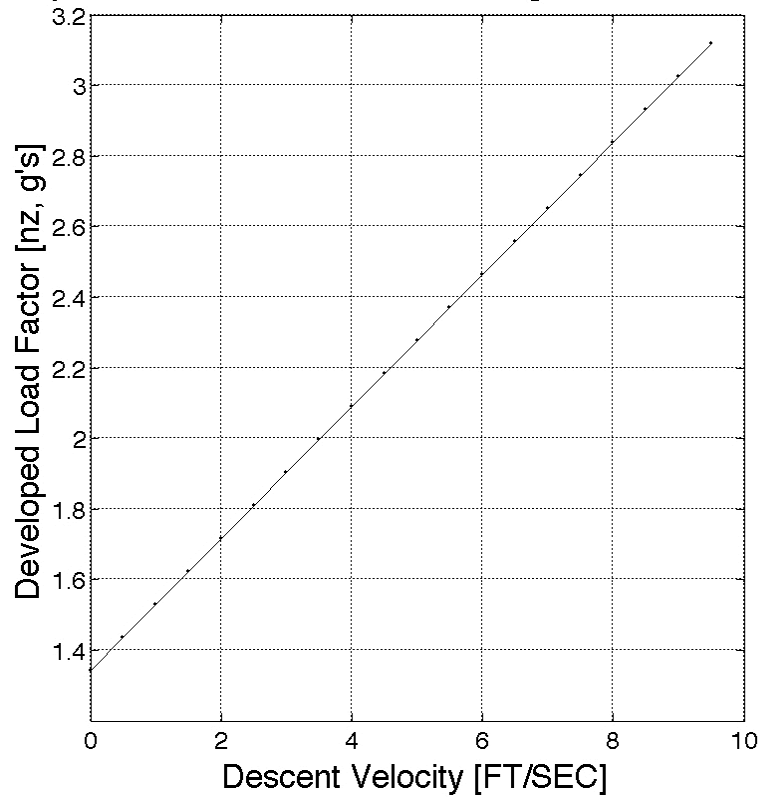


Figure 3. Landing Gear Drop Test Data

Finally, the alternating and mean stress values for landing and rebound can be calculated, the life read from Figure 2, and the damage determined.



## Taxi Loads

### APPENDIX 1. FLIGHT AND GROUND LOAD SPECTRA (CONTINUED)

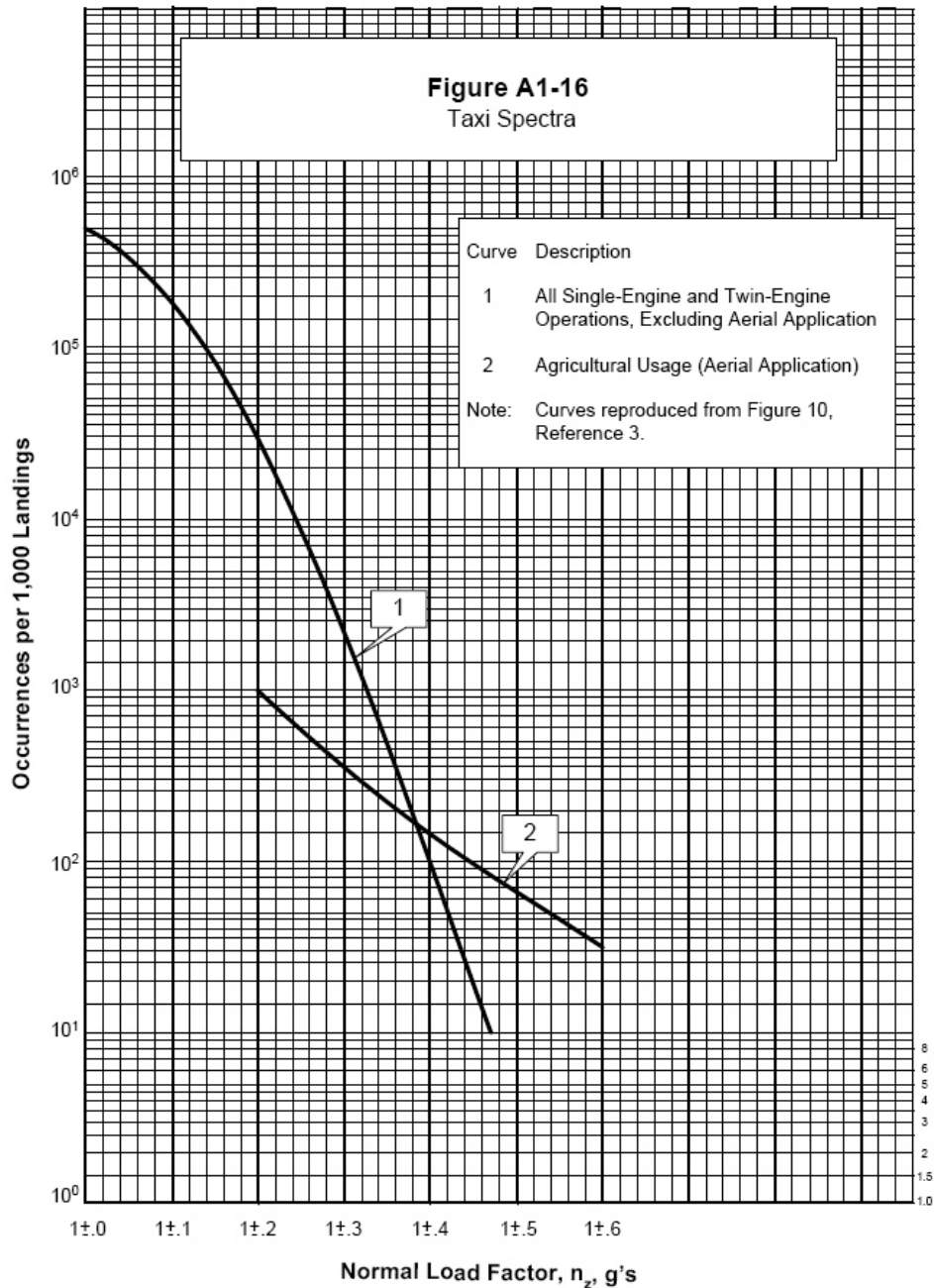


Figure 4. Exceedance Curve for Taxi

Using Figure 4 and following a similar procedure as explained previously for gust and maneuver, SMART|LD chooses different values of the taxi normal load factor doing a sweep through the spectrum. For this example, we chose four values: 0.0, 0.05, 0.1, and 0.15 in order to illustrate this method. (See row one in Table 2).

The damage calculations for the four examples values are shown in Table 2.



Taxi Damage					
1	From Figure 4 we make a sweep reading the $n_z$ ( $\pm$ ), Positive values of taxi normal load factor	0	0.05	0.1	0.15
		0.05	0.1	0.15	0.20
2	Calculate the average value from the values in row 1	0.025	0.075	0.125	0.175
3	From Figure 4 we read the different values of cumulative occurrence of taxi per 1,000 landings.	$6.4 \times 10^5$	$4.94 \times 10^5$	$3.35 \times 10^5$	$1.94 \times 10^5$
4	Frequency per 1,000 landings is the difference between two successive values in row 3.	$1.46 \times 10^5$	$1.59 \times 10^5$	$1.41 \times 10^5$	$1.03 \times 10^5$
5	Number of taxi cycles accumulated per flight.	$1.46 \times 10^2$	$1.59 \times 10^2$	$1.41 \times 10^2$	$1.03 \times 10^2$
6	Maximum and minimum Delta Stress over and below the Ground Stress (1g_Stress) – multiply row 2 by Ground Stress (-4520 psi)	-113	-339	-565	-791
		113	339	565	791
7	Mean stress (psi), and is calculated as in Equation 2	-4520	-4520	-4520	-4520
8	Alternating stress (psi) calculation – calculated as in Equation 3	113	339	565	791
9	Endurance cycles, N, read from Figure 2 (S-N diagram)	$1.90 \times 10^{16}$	$7.827 \times 10^{13}$	$6.09 \times 10^{12}$	$1.13 \times 10^{12}$
10	Damage per hour, row (5) divided by row (9)	$7.68 \times 10^{-15}$	$2.03 \times 10^{-12}$	$2.32 \times 10^{-11}$	$9.11 \times 10^{-11}$

Table 2. Taxi Damage Calculation

The total damage due to gust loading is the sum of the last row (Row 10) in Table 2, in this case:  $7.68 \times 10^{-15} + 2.03 \times 10^{-12} + 2.32 \times 10^{-11} + 9.11 \times 10^{-11} = 1.163 \times 10^{-10}$  per flight.

## Ground-Air-Ground Loads

Ground-air-ground (GAG) is a cycle which is defined by the transition from the minimum ground stress to the maximum stress during a complete flight. The transition from minimum flight or ground loading condition to maximum flight or ground loading condition during a flight is referred to as the once per flight ground-air-ground cycle. A schematic for the ground-air-ground cycle can be observed in Figure 5.

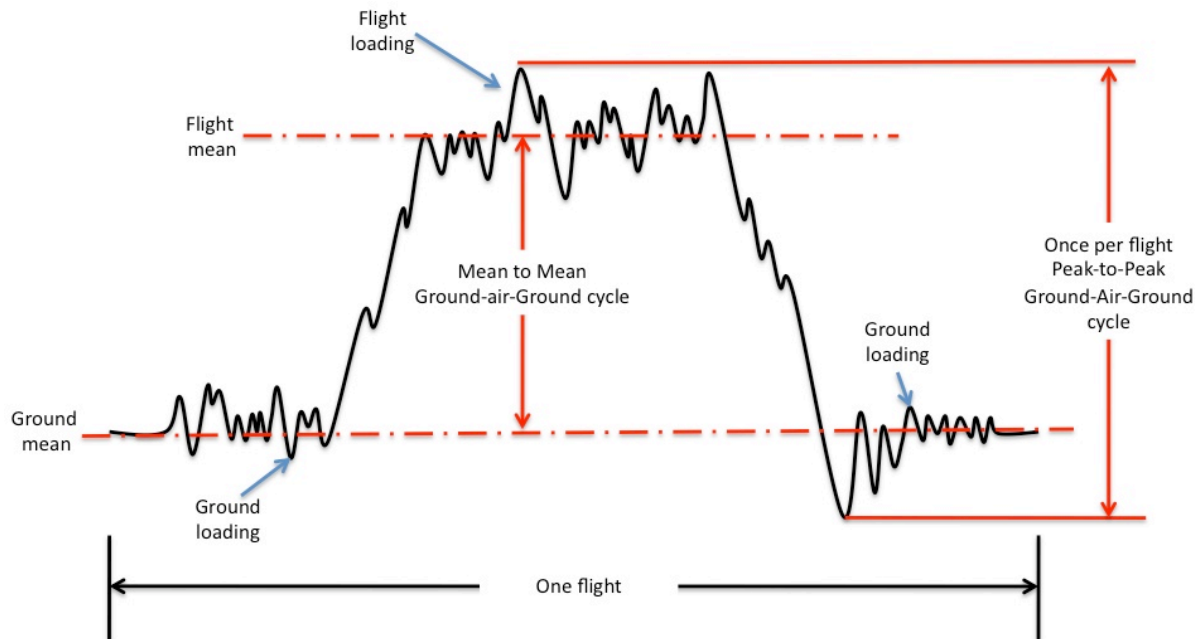


Figure 5. Ground-Air-Ground Cycle Schematic

SMART|LD uses the once-per-flight Peak-to-Peak value to calculate the Ground-Air-Ground cycle. The procedure used for this task is explained as follows:

- Load the maximum stresses for gust and maneuver including the corresponding number of occurrences per flight calculated from the exceedance curves.
- Calculate max\_GAG\_stress; this variable is developed by extracting the minimum value of gust and maneuver, the maximum value of gust and maneuver, and generating evenly spaced values of stress between the smallest and largest values. An example is shown on Table 4.
- Interpolate using the number of occurrences for maneuver and gust to determine the number of occurrences per flight for those evenly divided values of stress.
- Having the number of occurrences per flight at those stress levels for maneuver (shown as a red line in Figure 6) and gust (shown as a blue line Figure 6), add the number of occurrences in gust and number of occurrences in maneuver to obtain the total number of occurrences at each

max\_GAG\_stress level (shown as a green line in Figure 6).

- Since we are computing our damage calculations per flight, look for the max\_GAG\_stress that exceeds once per flight; this value is determined by interpolating max\_GAG\_stress and total number of occurrences at the cumulative frequency of once per flight (shown as an orange line in Figure 6).
- Min\_GAG\_stress is taken from landing, when the minimum stress occurs.
- Having the load pair, the life can be read from Figure 2, and the damage for GAG is calculated.

<b>Gust Min. Value (psi)</b>	9,000	<b>Minimum value</b>	9,000
<b>Man. Min. Value (psi)</b>	11,000		
<b>Gust Max. Value (psi)</b>	23,000	<b>Maximum value</b>	23,000
<b>Man. Max. Value (psi)</b>	21,000		
<b>Evenly divide values stress between the smallest and largest values</b>			
9,000			
11,000			
13,000			
15,000			
17,000			
19,000			
21,000			
23,000			

Table 3. Ground-Air-Ground Example

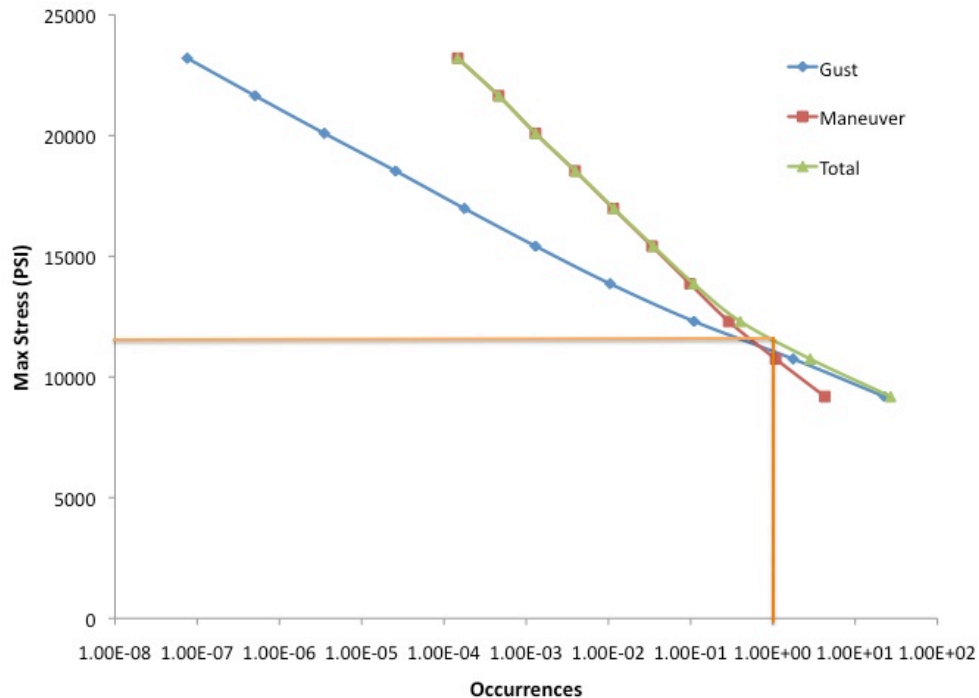


Figure 6. Ground-Air-Ground Example

Finally, all the damage for the different stages is added and the damage per hour or per flight calculated. Failure is assumed to occur when the Miner's coefficient reaches the critical value.